belonging to the casing of the screw; L_t is the part of the boundary D_t belonging to the screw; $\overline{\chi}(x, y, z)$ is a point in the space; $\overline{V}(\overline{\chi}, t)$ is the flow velocity vector at the point $\overline{\chi}$ at the time t; $\varphi_1(I_2)$, $\varphi_2(I_2)$ are the material functions of a Reiner–Rivlin liquid and characterize the effective and transverse viscosity; B(V) is the first White–Metzner cinematic tensor (strain rate tensor); P is the pressure; $I_2(\overline{V})$ is the second invariant of the strain rate tensor; ρ is a constant of the liquid; $\overline{N}_K(\overline{\chi})$ is the unit inner normal vector to K_t at the point $\overline{\chi} \in D_t$; $\overline{N}_L(\overline{\chi})$ is the unit outer normal vector to L_t at the point $\overline{\chi} \in L_t$; $pr_N^{\perp}$ is the orthogonal projection on a surface perpendicular to N; w is the angular rotational velocity of the screw; T designates transposition; and R_t is the matrix of rotation over a time t with angular velocity w around the axis of the spiral.

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SMALL-ASPECT TOMOGRAPHY OF HEATED FLOWS BASED ON IR-RADIOMETRIC MEASUREMENTS

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An algorithm for performing tomographic analysis of nonuniform heated gas flows is proposed. The algorithm employs infrared radiometric measurements and takes into account the real line structure of the vibrational–rotational bands of gases, including reabsorption.

The radiation emitted from a flow of heated gases contains rich information about the internal thermodynamic properties of the flow. It is natural to develop optical methods of diagnostics of such flows, especially since they have a number of significant advantages, including the fact that the diagnostics is performed remotely and does not disturb the medium under study. Maximum intensity of equilibrium thermal emission at a temperature of the order of 1000 K lies in the infrared region of the spectrum. Vibrational–rotational bands of many molecular gases lie in the same region. For this reason, in order to determine the temperature and concentration of the emitting components it is best to employ measurements in the IR region of the spectrum.

Methods for performing diagnostics of a uniform layer, which are based on measurements of the absorption coefficient and brightness of the radiation emitted by the layer in different spectral regions, are described in a number of works [1-3]. However, they are not applicable for diagnostics of flows which have significant spatial nonuniformity. The methods of computer tomography are widely employed to investigate nonuniform spatial struc-

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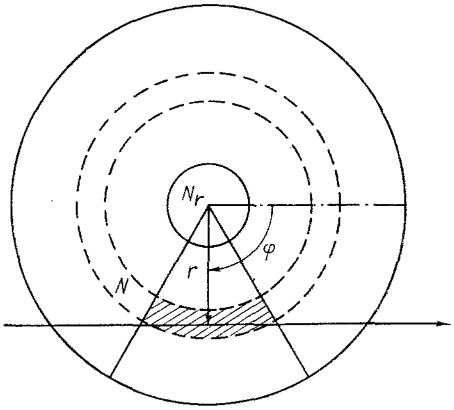

Fig. 1. Scheme used for partitioning the flow.

tures [4, 5]. The classical tomographic approach is based on analysis of the structure of the absorption field and does not take into account the possibility of characteristic emission of each element of the object. Taking into account the characteristic emission changes fundamentally both the technical implementation and the mathematical formulation of the method. As regards the IR region of the spectrum, in [6] the parameters of nonuniform flow are determined taking into account the characteristic emission and reabsorption. In [6] an algorithm is based on the method of successive approximations. However the real spectral structure of the radiation is not studied.

In this work we make an attempt to combine the two approaches described above. An algorithm is presented for determining the spatial distribution of the temperature and concentrations of the emitting components in a nonuniform flow taking into account the real structure of the radiation in the vibrational–rotational bands of molecular gases. As a rule, most molecular-gas flows studied are axisymmetric, and for this reason here we shall employ a cylindrical coordinate system (Fig. 1). The spectral density of energy brightness (SDEB) for aspect angle φ and impact parameter r can be rewritten, neglecting scattering, in the form

$$I_\nu(r,\ \varphi) = I_\nu^0 \exp\left(-\int_0^L \sum_i \chi_{i\nu}(T(x))\, C_i(x)\, dx\right) + \\ + \int_0^L B_\nu^0(T(x)) \sum_i \exp\left(-\int_0^X \sum_i \chi_{i\nu}(T(x'))\, C_i(x')\, dx'\right) \chi_{i\nu}(T(x))\, dx, \tag{1}$$

where x is the coordinate measured along the straight line from the extreme point of the flow to the radiation detector. The experimentally measured quantity is the SDEB, averaged over some range of frequencies, ususally $\Delta\nu \sim 10\ \text{cm}^{-1}$, determined by the transmission band of a selective element (IR filter, etc.):

$$I(r,\ \varphi) = \frac{1}{\Delta\nu} \int_{\nu-\frac{\Delta\nu}{2}}^{\nu+\frac{\Delta\nu}{2}} T(\nu')\, I_{\nu'}(r,\ \varphi)\, d\nu', \tag{2}$$

where $T(\nu)$ is the instrumental function of the radiation detector.

The line structure of the vibrational–rotational bands of molecules emitting IR radiation creates significant difficulties when calculating radiation transfer. The averaging interval $\Delta\nu$ contains a large number of spectral lines, each of which has its own structure. For this reason, in practice, instead of solving the spectral problem (1) and (2), different methods, based on a model of the real structure of spectral lines and approximate description of overlapping of the lines, are employed, to calculate radiation transfer. In this work we employ the following approach, which is an extension of the Curtis–Godson method [10] for strongly nonuniform paths. All lines falling within the chosen spectral interval $\Delta\nu$ fall into several groups, each of which contains a fixed number of spectral lines with the same half-width γ and temperature-dependent line strength. In order to calculate the SDEB along a ray the following expression is obtained instead of Eqs. (1) and (2):

$$I(r, \varphi) = I^0 R(L) + \int_0^L B^0(x)\, dR(x), \tag{3}$$

where the transmission of the layer 0–X is:

$$R(x) = \exp\left[-\sum_i \sum_{j=1}^{M} \frac{W_{i,j}}{\sqrt{1 + \frac{1}{4}\frac{W_{ij}^2}{V_{ij}}}}\right]; \tag{4}$$

$$W_{i,j} = \int_0^X AN_{i,j} AM_{i,j} \exp\left[-\frac{EE_{i,j}}{T(x)}\right] C_i dx;$$

$$V_{i,j} = AN_{i,j} \int_0^X \gamma AN_{i,j} AM_{i,j} \exp\left[-\frac{EE_{i,j}}{T(x)}\right] C_i dx.$$

Here M is the number of groups; $AN_{i,j}$, $AM_{i,j}$, $EE_{i,j}$ are the group parameters, determined for the i-th gas and a specific selective element based on a given (experimentally or computationally) dependence of the spectral absorption coefficient on the temperature and absorbing mass.

The inverse problem of determining the temperature and concentration fields reduces to solving the integral equation (3). The algorithm for numerical solution consists of the following. The flow is divided into N_r annular zones by means of concentric circles and into N_φ angles by means of rays emanating from the center of the flow (Fig. 1). Within one annular zone and one angle the flow parameters are assumed to be fixed. If the flow has axial symmetry, the parameters of the flow are fixed within each annular zone. In accordance with the principle of computer tomography and for convenience in obtaining the solution, the division into zones is performed in a manner so that the boundaries of the annular zones pass through points located midway between successive sections, in which measurements of the SDEB are performed, and the aspects are taken at angles separated by identical spacings $\Delta\varphi = 2\pi/N_\varphi$.

We shall first study the solution of the problem (3) for an axisymmetric flow. The quantity SDEB, measured in the first ray $I_{1,\nu}$, passing through the extreme annular zone of the flow, is determined by the concentrations $C_1{}^i$ of the emitting components i in the first zone and the temperature T_1 in the first zone:

$$I_{1,\nu} = F_{1,\nu}(C_1^i, T_1). \tag{5}$$

Correspondingly, for the section passing through the N-th annular zone, we can write

$$I_{N,\nu} = F_{N,\nu}(C_1^i, T_1, C_2^i, T_2, \ldots, C_N^i, T_N). \tag{6}$$

Equation (5) contains i + 1 unknowns, so that at least i + 1 equations are required to obtain a solution. This corresponds to measurements of SDEB at $K \geq i + 1$ wavelengths. In order to determine the parameters $C_1{}^i$ and T_1 in the first zone we obtain a system of K nonlinear equations. Solving the system of equations (5) and substituting the parameters found into the system of equations (6) we find successively the parameters in the N-th zone, where N varies from 2 to N_r. After this the values of the parameters C_i and T in all annular zones are determined. If the flow is not axisymmetric, the SDEB is measured for all sections at different aspect angles. In this case, systems of equations for all aspect angles are added to the system of equations (5) and (6) for each zone. The values of the parameters of the flow in each annular zone within each angle φ are determined by solving successively the system of equations for all aspect angles in all zones. The generalized Newton's method and SVD factorization [11] were employed to implement numerically the above algorithm for solving the systems of nonlinear equations (5) and (6).

The basic difficulties in solving problems in computer tomography stem from the possible instability of the obtained solution. There are additional problems connected with the need to choose optimal parameters of the recording instruments and conditions for performing the experiment in order to minimize the interference due to noise, absorption by the intervening atmosphere, etc., on the accuracy with which C_i and T in the flow are determined.

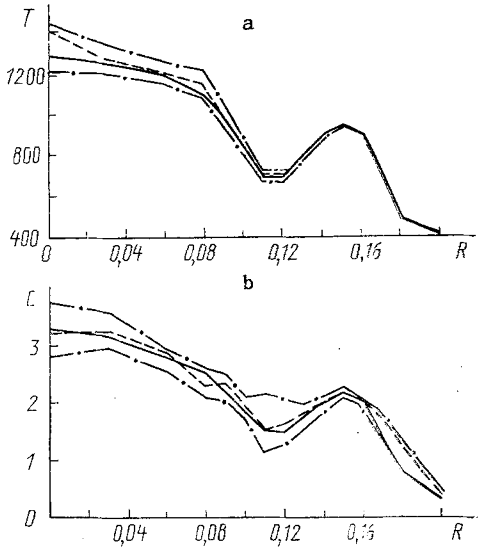

Fig. 2. Comparison of fixed (solid curves) and reconstructed (dashed curves) radial distributions of the temperature (a) and CO_2 concentration (b). The dot–dashed line shows the confidence interval. T, K; C, %; R, m.

These problems are best solved by using mathematical modeling, i.e., computer simulation of all significant operations performed by the measuring instrument, as well as by giving a formal description of the object of interest and of the physical processes. By performing all this on a computer, we can determine the optimal arrangement of the experiment, the number and parameters of IR light filters in the measuring apparatus, etc. The numerical-testing scheme is as follows. Some field of temperatures and concentrations of the emitting components in the flow is given. Then the SDEB of the flow, which are recorded by the detector at different wavelengths (which are determined by the IR filters employed) in different sections and at different aspect angles taking into account the intervening atmosphere and the instrumental function of the detector, are calculated. A perturbation, distributed according to a normal law with some fixed mean relative error, is superposed on the values obtained for the SDEB. The transverse profiles of SDEB distorted in this manner are employed for solving the tomographic problem. The fields of the arithmetic-mean values of C_i and T and the standard deviations are determined by random selection of variants of the solutions.

The method described above was implemented in a program that makes it possible to determine the spatial distributions of the temperature and concentrations of CO_2, H_2O, CO, HCl, and soot in a flow from the transverse SDEB profiles measured at different wavelengths. The characteristic computing time for a 20-band approximation and two components in the flow is equal to approximately 1 min on a PC/AT 386 computer.

As an example Fig. 2 gives a variant of the calculation using the above-described method for axisymmetric gas flow, containing CO_2. Two light filters with transmission maxima at the wavelengths $\lambda_1 = 2.68$ and $\lambda_2 = 4.44$ μm with relative half-widths of 2% were employed for diagnostics. The detector was placed at a distance of 100 cm from the boundary of the flow. The atmosphere was assumed to be standard. The flow was divided into 15 zones. The error in the recorded values of the SDEB was modelled by superposing on the values obtained from the direct calculation for each section a random perturbation with average relative error of 3%. The standard deviations are maximum at the center of the flow and are equal to 10% for the temperature and 15% for the CO_2 concentration. When the random error in the values of SDEB was reduced to 1%, the maximum errors in determining the temperature and concentration dropped to 2 and 5%, respectively. It should be noted, however, that in reality the computational error can be higher because of the uncertainty in the optical characteristics of the emitting components.

The program was tested for a large number of different variants of spatial distributions and different components in the flow in the temperature range 300-2500 K and it showed that the stability of the solution is good and the concentration and temperature fields can be reconstructed from measurements in 10-20 projections using one to eight (for asymmetric flows) aspect angles.

Thus the proposed method makes it possible to perform tomographic analysis of nonuniform flows based on IR-radiometric measurements taking into account the real line structure of vibrational–rotational bands of gases, taking into account reabsorption. The collection of gases for which diagnostics can be performed can be enlarged if necessary. An analogous approach is also applicable for diagnostics of other flow characteristics, affecting the emissive properties of the flow, for example, vibrational nonuniformity.

NOTATION

Here r is the impact parameter; φ is the aspect angle; ν is the frequency; $I_\nu(r, \varphi)$ is the spectral density of the energy brightness; $I_\nu{}^0$ is the background emission; C_i and χ_i are the concentration and spectral absorption coefficients of the i-th component; $B_\nu{}^0$ is Planck's function; T is the temperature; $T(\nu)$ is the instrumental function of the detector; R(x) is the transmission of the layer 0–X; $AN_{i,j}$, $AM_{i,j}$, $EE_{i,j}$ are the parameters of the j-th group of the i-th component; γ is the half-width of the line; N_r is the number of annular zones; N_φ is the number of aspect angles; $F_{N,\nu}$ is a functional of the concentrations and temperatures in each of the N zones; and K is the number of filters.

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